# BDC

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# Cultural landscapes: evaluating for managing the change

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## FUZZY LOGIC AND SPATIAL ANALYSIS IN GIS ENVIRONMENT

*Ferdinando Di Martino, Salvatore Sessa*

**Abstract**

In the context of the fuzzy logic we use a system of max-min fuzzy relation equations to solve a problem of spatial analysis in a Geographical Information Systems (GIS). The geographical area under study is divided in subzones to which we apply our process to determine the outputs after that an expert sets the whole SFRE with the values of the coefficients impacting the input data. We find the best solutions by associating the results to each subzone and thematic maps are extracted from the GIS.

Keywords: system of max-min fuzzy relation equations, GIS, triangular fuzzy number

## FUZZY LOGIC E ANALISI SPAZIALE IN AMBIENTE GIS

**Sommario**

Nell’ambito della logica fuzzy si propone un sistema di equazioni di relazioni fuzzy (SFRE) max-min per risolvere un problema di analisi spaziale in un Geographical Information System (GIS). L’area geografica di studio viene divisa in subzone a cui si applica l’approccio elaborato. I risultati ottenuti tenngono conto del punto di vista di un esperto che ha fissato, per l’intero SFRE, i valori dei coefficienti che influenzano i dati di input. Le soluzioni migliori sono state determinate associando i risultati ad ogni subzona, per cui sono state elaborate delle opportune mappe tematiche in GIS.

Parole chiave: sistema di equazioni con relazione fuzzy max-min, GIS, numero fuzzy triangolare

## 1. Introduction

A Geographical Information System (GIS) is used to analyze spatial distribution of data and simple examples of this analysis are the creation of thematic maps. Often the decision maker is obliged to use a GIS for integrating a huge mass of data as images, spatial layers, attributes information and afterwards he must utilize an inference mechanism based on these attributes. The diversity and the inhomogeneity between these data can lead to uncertain decisions, so that one recurs to fuzzy logic to handle these uncertain information (Di Martino *et al.*, 2005a; Di Martino *et al.*, 2005b; Di Martino *et al.*, 2008; Groenemans *et al.*, 1997; Hemetsberger *et al.*, 2002). Here we propose an inferential method based on the resolution of a system of fuzzy relation equations (shortly, SFRE) applied in a GIS environment. Usually a SFRE with max-min composition is read as:

$$\begin{cases} (a_{11} \wedge x_1) \vee ... \vee (a_{1n} \wedge x_n) = b_1 \\ (a_{21} \wedge x_1) \vee ... \vee (a_{2n} \wedge x_n) = b_2 \\ ... \\ (a_{m1} \wedge x_1) \vee ... \vee (a_{mn} \wedge x_n) = b_m \end{cases} \tag{1}$$

The system (1) is consistent (i.e. has solutions) if and only if it has the greatest solution, moreover it has minimal solutions (Chen and Wang, 2002; De Baets, 2000; Di Nola *et al.*, 1989; Higashi and Klir, 1984; Li and Fang, 2009; Sanchez, 1976).

We schematize in Fig. 1 the process here used and in the sequel summarized:

- the input data are extracted and stored in the dataset;
- a fuzzy partition of the input domain is made by means of triangular fuzzy numbers (TFN);
- the membership degrees of each TFN determine the coefficients $\{b_1,\dots,b_m\}$ of (1). The coefficients $a_{ij}$ are fixed by the expert and all the solutions $(x_1,\dots,x_n)$ of (1) is determined;
- a fuzzy partition of [0,1] is created for the output variables $o_1,\dots,o_k$; every TFN of the partition corresponds to a determined value $x_j$;
- the output data $o_1,\dots,o_k$ are extracted and the linguistic label of the most appropriate fuzzy set, represented by a TFN, is assigned to the output variable $o_j$.

For sake of completeness, we recall that a TFN is a continuous and fuzzy-convex real function μ: R→[0,1], for which there exist three real numbers a,b,c, such that μ(x)=0 for x outside [a,b], μ(c)=1 for an unique point c (usually it can be considered as the midpoint) between a and b, μ is non-decreasing in [a,c] and non-increasing in [c,b].

The expert applies the SFRE (1) on each subzone. The input data are the symptoms, the parameters to be determined are the causes. For example, let us consider a planning problem. A city planner determines in each subzone the mean state of buildings ($x_1$) and the mean soil permeability ($x_2$), knowing the number of collapsed building in the last year ($b_1$) and the number of flooding in the last year ($b_2$).

The expert creates the system (1) for each subzone by setting the impact matrix A, whose entries $a_{ij}$ (i=1,…,n and j=1,…,m) represent the impact of the j-th cause $x_j$ to the production of the i-th symptom $b_i$.

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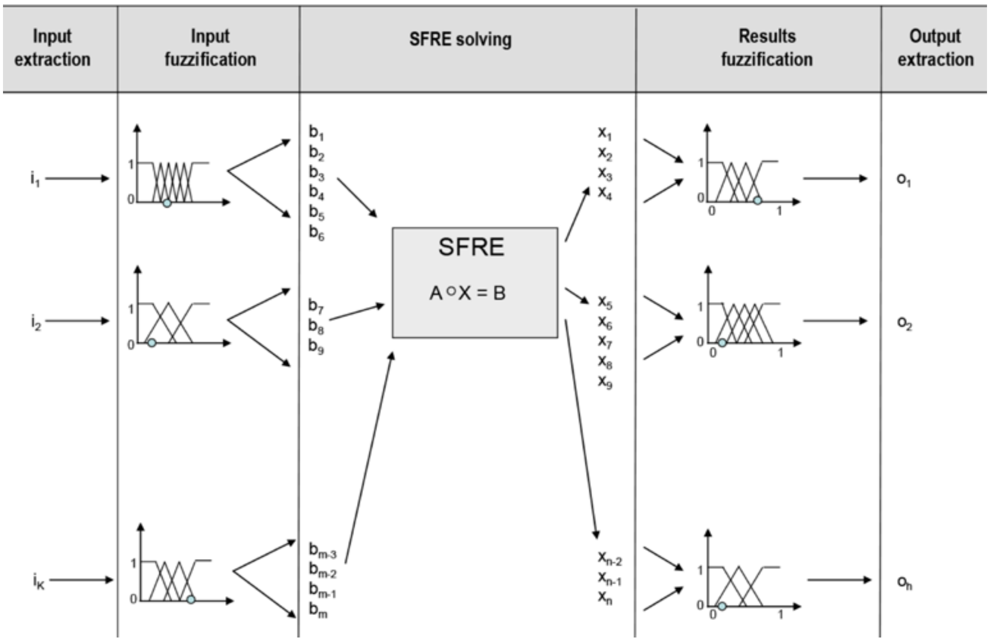
For example, we consider the equation:

$(0.8 \wedge x_1) \vee (0.2 \wedge x_2) \vee (0.0 \wedge x_3) \vee (0.8 \wedge x_4) \vee (0.3 \wedge x_5) \vee (0.0 \wedge x_6) = b_3 = 0.9$

the expert gives for the symptom $b_3$ = “*collapsed building in the last year* = *high*”= 0.9, an impact 0.8 of the variable “*mean state of buildings=scanty*” or an impact 0.2 of the variable “*mean state of buildings* = *medium*” or an impact 0.0 of the variable “*mean state of buildings* = *high*” or an impact 0.8 of the variable “*mean soil permeability* = *low*” or an impact 0.3 of the variable “*mean soil permeability* = *medium*” or an impact 0.0 of the variable “*mean soil permeability* = *high*”.

**Fig. 1 – Resolution process of a SFRE**




We can determine the maximal interval solutions of (1). Each maximal interval solution is an interval whose extremes are the values taken from a lower solution and from the greatest solution. Every value $x_i$ belongs to this interval. If the SFRE (1) is inconsistent, it is possible to determine the rows for which no solution is permitted. If the expert decides to exclude the row for which no solution is permitted, he considers that the symptom $b_i$ (for that row) is not relevant to its analysis and it is not taken into account.
Otherwise, the expert can modify the setting of the coefficients of the matrix A to verify if the new system has some solution. In general, the SFRE (1) has T maximal interval solutions $X_{max(1)},\ldots,X_{max(T)}$. In order to describe the extraction process of the solutions, let

.

$X_{max(t)}$, $t \in \{1,\ldots,T\}$, be a maximal interval solution given below, where $X^{low}$ is a lower solution and $X^{gr}$ is the greatest solution. Our aim is to assign the linguistic label of the most appropriate fuzzy sets corresponding to the unknown $\{x_{j_1}, x_{j_1}, \ldots, x_{j_s}\}$ related to an output variable $o_s$, s = 1,…,k. For example, assume that the three fuzzy sets $x_1$, $x_2$, $x_3$ (resp., $x_4$, $x_5$, $x_6$) are related to $o_1$ (resp., $o_2$) and are represented from the TFNs given in Table 1, where INF(j), MEAN(j), SUP(j) are the three fundamental values of the generic TFN $x_j$ , $j=j_1, \ldots, j_s$. We can write their membership functions $\mu_{j_1}, \mu_{j_2}, \ldots, \mu_{j_h}$ as follows:

$$\mu_{j_1} = \begin{cases} 1 & \text{if } \mathrm{INF}(j_1) \le x \le MEAN(j_1) \\ \dfrac{SUP(j_1) - x}{SUP(j_1) - MEAN(j_1)} & \text{if } MEAN(j_1) < x \le SUP(j_1) \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

$$\mu_{j} = \begin{cases} \dfrac{x - INF(j)}{MEAN(j) - INF(j)} & \text{if } \mathrm{INF(j)} \le x \le MEAN(j) \\ \dfrac{SUP(j) - x}{SUP(j) - MEAN(j)} & \text{if } \mathrm{MEAN(j)} < x \le SUP(j) \text{ and } j \in \{j_2, \ldots, j_{s-1}\} \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

$$\mu_{j_s} = \begin{cases} \dfrac{x - INF(j_s)}{MEAN(j_s) - INF(j_s)} & \text{if } INF(j_s) \le x \le MEAN(j_s) \\ 1 & \text{if } MEAN(j_s) < x \le SUP(j_s) \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

**Tab. 1 – TFNs values for the fuzzy sets**

| Unknown | INF(j) | MEAN(j) | SUP(j) |
|---|---|---|---|
| $x_1$ | 0.0 | 0.2 | 0.4 |
| $x_2$ | 0.3 | 0.5 | 0.7 |
| $x_3$ | 0.6 | 0.8 | 1.0 |
| $x_4$ | 0.0 | 0.2 | 0.4 |
| $x_5$ | 0.3 | 0.5 | 0.7 |
| $x_6$ | 0.6 | 0.8 | 0.1 |

.

If $XMin_t(j)$ (resp. $XMax_t(j)$) is the min (resp., max) value of every interval t(j) = [$XMin_t(j)$, $XMax_t(j)$] corresponding to the unknown $x_j$, we can calculate the arithmetical mean value $XMean_t(j)$ of the j-th component of the above maximal interval solution $X_{max(t)}$ as

$$XMean_t(j) = \frac{XMin_t(j) + XMax_t(j)}{2} \tag{5}$$

and we get the vector column $XMean_t$=[$XMean_t(1)$,…, $XMean_t(n)$]$^{-1}$ (Tab. 2). The value given from max{$XMean_t(j_1)$,…,$XMean_t(j_s)$} obtained for the unknowns $x_{j_1},\dots,x_{j_s}$ corresponding to the output variable $o_s$, is the linguistic label of the fuzzy set assigned to $o_s$ and it is denoted by $score_t(o_s)$, defined also as reliability of $o_s$ in the interval solution t(j). For the output vector O = [$o_1$,…,$o_k$] we define the following reliability index in the interval solution t as:

$$\mathrm{Re}\,l_t(O) = \frac{1}{k}\cdot\sum_{s=1}^{k} score_t(o_s) \tag{6}$$

and then as final reliability index of O, the number Rel(O)=max{$Rel_t(O)$: t=1,…,T}.

**Tab. 2 – TFNs mean values**

| Output variable | Unknown component | Linguistic label | XMint(j) | XMaxt(j) | XMeant(j) |
|---|---|---|---|---|---|
| $o_1$ | $x_1$ | scanty | 0.6 | 0.8 | 0.70 |
| | $x_2$ | medium | 0.2 | 0.4 | 0.30 |
| | $x_3$ | good | 0.0 | 0.1 | 0.05 |
| $o_2$ | $x_4$ | low | 0.3 | 0.5 | 0.40 |
| | $x_5$ | medium | 0.4 | 0.7 | 0.55 |
| | $x_6$ | good | 0.0 | 0.3 | 0.15 |

In Section 2 we give an overview on the determination of the set of the solutions of a SFRE and in Section 3 we show how the proposed algorithm is applied in spatial analysis. Section 4 contains the results of our simulation.

## 2. An overview of SFRE

We have the following known form of (1):

$$A \circ X = B \tag{1}$$

.

where A = ($a_{ij}$), is the matrix of coefficients, X = $(x_1,x_2,\ldots,x_n)^{-1}$ is the column vector of the unknowns and B = $(b_1,b_2,\ldots,b_m)^{-1}$ is the column vector of the known terms, being $a_{ij}$, $x_j$, $b_i$ $\in$ [0,1] for each i = 1,…,m and j = 1,…,n.

We have the following definitions and terminologies: the whole set of all solutions X of the SFRE (7) is denoted by $\Omega$. If $\Omega \neq \emptyset$, then the SFRE (7) is called consistent, otherwise it is called inconsistent. A solution $\hat{X} \in \Omega$ is called a lower (or minimal) solution if $X \leq \hat{X}$ for some $X \in \Omega$ implies X= $\hat{X}$ , where "≤" is the partial order induced in $\Omega$ from the natural order of [0, 1]. If the lower solution is unique, then it is the least (or minimum) solution of the SFRE (7). We also recall that the system (7) has the unique greatest (or maximum) solution $X^{gr} = (x_1^{gr}, x_2^{gr}, \ldots, x_n^{gr})^{-1}$ if and only if $\Omega \neq \emptyset$ [23]. A maximal interval solution of (7) is of the following type:

$$X_{\max(t)} = \begin{pmatrix} [a_1, x_1^{gr}] \\ [a_2, x_2^{gr}] \\ [\ldots, \ldots] \\ [a_n, x_n^{gr}] \end{pmatrix}$$

where $[a_j, x_j^{gr}] \subseteq [0,1]$ if $a_j$ is a membership value of a lower solution and every X=$(x_1,x_2,\ldots,x_n)^{-1}$ in $\Omega$ is such that $x_j \in [a_j, x_j^{gr}]$ for each j = 1,…,n (t varies from 1 till to the number of lower solutions).

In order to determine if a SFRE is consistent, we have used the universal algorithm of Peeva and Kyosev (2004) based on the above concepts. This algorithm has been implemented and tested under C++ language.

The C++ library has been integrated in the ESRI ArcObject Library of the tool ArcGIS 9.3 for a problem of spatial analysis illustrated in Section 3.

**3. SFRE in spatial analysis**

We consider a specific area of study on the geographical map on which we have a spatial data set of "causes" and we want to analyze the possible "symptoms". We divide this area in P subzones where a subzone is an area in which the same symptoms are derived by input data, and the impact of a symptom on a cause is the same one as well. It is important to note that even if two subzones have the same input data, they can have different impact degrees of symptoms on the causes. For example, the cause that measures the occurrence of floods may be due with different degree of importance to the presence of low porous soils or to areas subjected to continuous rains. Afterwards the area of study is divided in homogeneous subzones, hence the expert creates a fuzzy partition for the domain of each input variable and, for each subzone, he determines the values of the symptoms $b_i$, as the membership degrees of the corresponding fuzzy sets (cfr. input fuzzification process of Fig. 1).

For each subzone, then the expert sets the most significant equations and the values $a_{ij}$ of impact of the j-th cause to the i-th symptom creating the SFRE (1). After the determination

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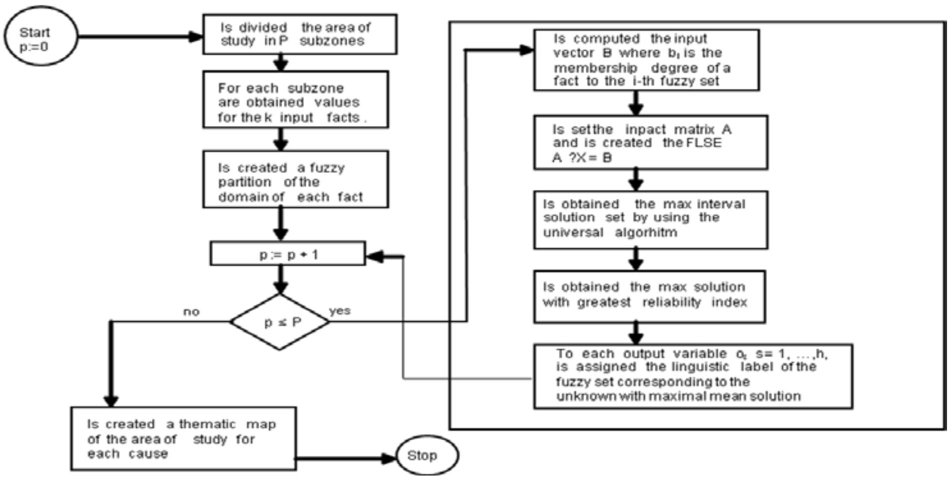
of the set of maximal interval solutions by using the algorithm of Section 2, the expert for each interval solution calculates, for each unknown $x_j$, the mean interval solution $XMean_t(j)$ with (5). The linguistic label $Rel_t(o_s)$ is assigned to the output variable $o_s$.
Then he calculates the reliability index $Rel_t(O)$, given from formula (6), associated to this maximal interval solution t. After the iteration of this step, the expert determines the reliability index (6) for each maximal interval solution, by choosing the output vector O for which Rel(O) assumes the maximum value. Iterating the process for all the subzones, the expert can show the thematic map of each output variable. We schematize the whole process in Fig. 2.
At the end of the process the user can create a thematic map of a specific output variable over the area of study and also a thematic map of the reliability index value obtained for the output variable. If the SFRE related to a specific subzone is inconsistent, the expert can decide whether or not eliminate rows to find solutions: in the first case he decides that the symptoms associated to the rows that make the system inconsistent are not considered and eliminates them, so reducing the number of the equations. In the second case, he decides that the correspondent output variable for this subzone remain unknown and it is classified as unknown on the map.

**Fig. 2 – The schematization of the methodological process**




## 4. Simulation results

Here we show the results of an experiment in which we apply our method to census statistical data agglomerated on four districts of the east zone of Naples (Fig. 3). We use the year 2000 census data provided by the Istituto Nazionale di Statistica (ISTAT).
These data contain information on population, buildings, housing, family, employment work for each census zone of Naples. Every district is considered as a subzone with homogeneous input data given in Table 4.

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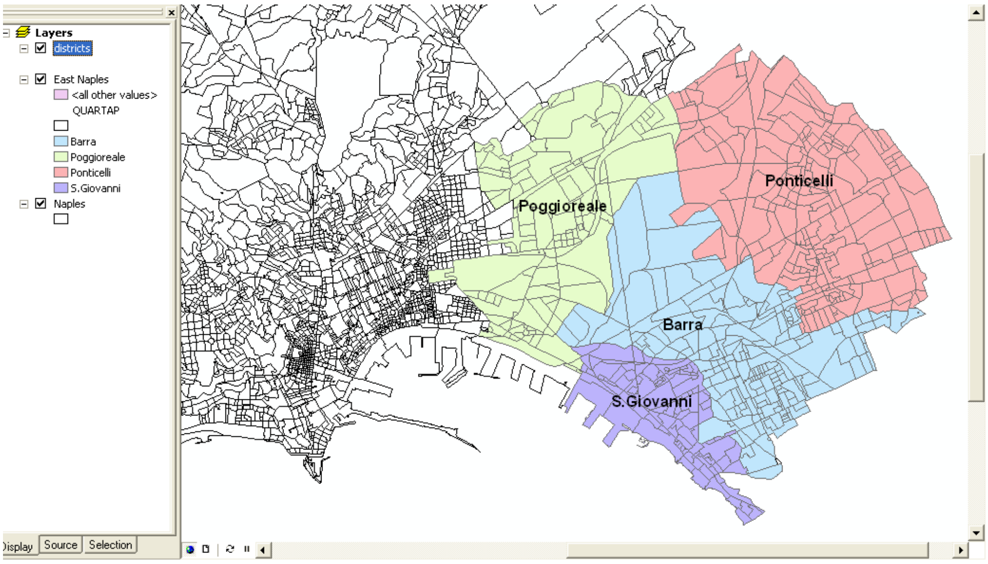
In this experiment we consider the following four output variables: "$o_1$ = *Economic prosperity*" (wealth and prosperity of citizens), "$o_2$ = *Transition into the job*" (ease of finding work), "$o_3$ = *Social Environment*" (cultural levels of citizens) and "$o_4$ = *Housing development*" (presence of building and residential dwellings of new construction).
For each variable we create a fuzzy partition composed by three TFNs called "low", "mean" and "high" presented in Table 3.
Moreover we consider the following seven input parameters:

- $i_1$=percentage of people employed=number of people employed/total work force; $i_2$=percentage of women employed=number of women employed/number of people employed;
- $i_3$=percentage of entrepreneurs and professionals = number of entrepreneurs and professionals/number of people employed;
- $i_4$ = percentage of residents graduated=numbers of residents graduated/number of residents with age > 6 years;
- $i_5$=percentage of new residential buildings=number of residential buildings built since 1982/total number of residential buildings;
- $i_6$ = percentage of residential dwellings owned=number of residential dwellings owned/ total number of residential dwellings;
- $i_7$ = percentage of residential dwellings with central heating system = number of residential dwellings with central heating system/total number of residential dwellings.

In Table 4 we show these input data for the four subzones.

**Fig. 3 – Area of study: four districts at East of Naples (Italy)**



For the fuzzification process of the input data the expert indicates a fuzzy partition for each input domain formed from three TFNs labeled "low", "mean" and "high", whose values are reported in Table 5. In Tables 6 and 7 we show the values obtained for the 21 symptoms $b_1,...,b_{21}$, moreover we report the input variable and the linguistic label of the correspondent TFN for each symptom $b_i$. In order to form the SFRE (1) in each subzone, the expert defines the equations by setting the impact values aij by basing over the most significant symptoms.

**Tab. 3 – Values of the TFNs low, mean, high**

| Output | low | | | mean | | | high | | |
|---|---|---|---|---|---|---|---|---|---|
| | INF | MEAN | SUP | INF | MEAN | SUP | INF | MEAN | SUP |
| $o_1$ | 0.0 | 0.3 | 0.5 | 0.3 | 0.5 | 0.8 | 0.5 | 0.8 | 1.0 |
| $o_2$ | 0.0 | 0.3 | 0.5 | 0.3 | 0.5 | 0.8 | 0.5 | 0.8 | 1.0 |
| $o_3$ | 0.0 | 0.3 | 0.5 | 0.3 | 0.5 | 0.8 | 0.5 | 0.8 | 1.0 |
| $o_4$ | 0.0 | 0.3 | 0.5 | 0.3 | 0.5 | 0.8 | 0.5 | 0.8 | 1.0 |

**Tab. 4 – Input data obtained for the four subzones**

| District | $i_1$ | $i_2$ | $i_3$ | $i_4$ | $i_5$ | $i_6$ | $i_7$ |
|---|---|---|---|---|---|---|---|
| Barra | 0.604 | 0.227 | 0.039 | 0.032 | 0.111 | 0.424 | 0.067 |
| Poggioreale | 0.664 | 0.297 | 0.060 | 0.051 | 0.086 | 0.338 | 0.149 |
| Ponticelli | 0.609 | 0.253 | 0.039 | 0.042 | 0.156 | 0.372 | 0.159 |
| S. Giovanni | 0.576 | 0.244 | 0.041 | 0.031 | 0.054 | 0.353 | 0.097 |

**Tab. 5 – TFNs values for the input domains**

| Input variable | low | | | mean | | | high | | |
|---|---|---|---|---|---|---|---|---|---|
| | INF | MEAN | SUP | INF | MEAN | SUP | INF | MEAN | SUP |
| $i_1$ | 0.00 | 0.40 | 0.60 | 0.40 | 0.60 | 0.80 | 0.60 | 0.80 | 1.00 |
| $i_2$ | 0.00 | 0.10 | 0.30 | 0.10 | 0.30 | 0.40 | 0.30 | 0.50 | 1.00 |
| $i_3$ | 0.00 | 0.04 | 0.06 | 0.04 | 0.06 | 0.10 | 0.07 | 0.20 | 1.00 |
| $i_4$ | 0.00 | 0.02 | 0.04 | 0.02 | 0.04 | 0.07 | 0.04 | 0.07 | 1.00 |
| $i_5$ | 0.00 | 0.05 | 0.08 | 0.05 | 0.08 | 0.10 | 0.08 | 0.10 | 1.00 |
| $i_6$ | 0.00 | 0.10 | 0.30 | 0.10 | 0.30 | 0.60 | 0.30 | 0.60 | 1.00 |
| $i_7$ | 0.00 | 0.10 | 0.30 | 0.10 | 0.30 | 0.50 | 0.30 | 0.50 | 1.00 |

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**Tab. 6 – TFNs for the symptoms $b_1 \div b_{12}$**

| Subzone | $b_1$: $i_1$ = low | $b_2$:$i_1$ =mean | $b_3$:$i_1$ =high | $b_4$:$i_2$ =low | $b_5$:$i_2$ =mean | $b_6$:$i_2$ =high | $b_7$:$i_3$ =low | $b_8$:$i_3$ =mean | $b_9$:$i_3$ =high | $b_{10}$: $i_4$=low | $b_{11}$: $i_4$=mean | $b_{12}$: $i_4$=high |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Barra | 0.00 | 0.98 | 0.02 | 0.36 | 0.63 | 0.00 | 1.00 | 0.00 | 0.00 | 0.40 | 0.60 | 0.00 |
| Poggioreale | 0.00 | 0.93 | 0.07 | 0.01 | 0.99 | 0.00 | 0.00 | 1.00 | 0.00 | 0.00 | 0.63 | 0.37 |
| Ponticelli | 0.00 | 0.91 | 0.05 | 0.23 | 0.76 | 0.00 | 1.00 | 0.00 | 0.00 | 0.00 | 0.93 | 0.07 |
| S. Giovanni | 0.12 | 0.88 | 0.00 | 0.28 | 0.72 | 0.00 | 0.95 | 0.05 | 0.00 | 0.45 | 0.55 | 0.00 |

**Tab. 7 – TFNs for the symptoms $b_{13} \div b_{21}$**

| **Subzone** | **$b_{13}$:$i_5$ =low** | **$b_{14}$:$i_5$= mean** | **$b_{15}$:$i_5$= high** | **$b_{16}$:$i_6$= low** | **$b_{17}$:$i_6$= mean** | **$b_{18}$:$i_6$= high** | **$b_{19}$:$i_7$= low** | **$b_{20}$:$i_7$= mean** | **$b_{21}$:$i_7$ =high** |
|---|---|---|---|---|---|---|---|---|---|
| Barra | 0.00 | 0.00 | 0.10 | 0.00 | 0.59 | 0.41 | 1.00 | 0.00 | 0.00 |
| Poggioreale | 0.00 | 0.70 | 0.30 | 0.00 | 0.87 | 0.13 | 0.75 | 0.25 | 0.00 |
| Ponticelli | 0.00 | 0.00 | 1.00 | 0.00 | 0.76 | 0.24 | 0.70 | 0.30 | 0.00 |
| S. Giovanni | 0.87 | 0.13 | 0.00 | 0.00 | 0.82 | 0.18 | 1.00 | 0.00 | 0.00 |

As example, we illustrate this procedure for the subzone "Barra". Similar procedures can be adopted for the other three remaining subzones.
For the subzone "Barra", the expert chooses the significant symptoms $b_2$, $b_4$, $b_5$, $b_7$, $b_{10}$, $b_{11}$, $b_{15}$, $b_{17}$, $b_{18}$, $b_{19}$, by obtaining a SFRE (7) with m = 10 equations and n = 12 unknowns.
The matrix A of the impact values $a_{ij}$ has sizes 10×12 and the vector B of the symptoms $b_i$ has sizes 10×1 and both are given below.
The SFRE (1) is inconsistent and eliminating the rows such that the equation becomes consistent, we obtain four maximal interval solutions $X_{max(t)}$ (t=1,…,4) and we calculate the vector column $XMean_t$ on each maximal interval solution. Hence we associate to the output variable $o_s$ (s = 1,…,4), the linguistic label of the fuzzy set with the higher value calculated with formula (5) obtained for the corresponding unknowns $x_{j_1},\ldots,x_{j_s}$ and given in Table 8. For determining the reliability of our solutions, we use the index given by formula (6). We obtain that $Rel_t(o_1) = Rel_t(o_2) = Rel_t(o_3) = Rel_t(o_4) = 0.6025$ for t=1,…,4 and hence $Rel(O)=\max\{Rel_t(O): t=1,\ldots,4\}=0.6025$ where $O=\{o_1,\ldots,o_4\}$.
We note that the same final set of linguistic labels associated to the output variables $o_1$ = "high", $o_2$ = "mean", $o_3$ = "low", $o_4$ = "low" is obtained as well. The relevant quantities are given below.

.

$$
\mathrm{A}=\begin{pmatrix}
0.5 & 1.0 & 0.0 & 0.4 & 1.0 & 0.2 & 0.2 & 0.7 & 0.3 & 0.1 & 0.3 & 0.2\\
0.3 & 0.5 & 0.2 & 0.4 & 0.5 & 0.4 & 0.3 & 0.6 & 0.2 & 0.0 & 0.0 & 0.0\\
0.2 & 0.7 & 0.2 & 0.2 & 0.7 & 0.2 & 0.2 & 0.7 & 0.2 & 0.0 & 0.0 & 0.0\\
1.0 & 0.2 & 0.0 & 0.8 & 0.3 & 0.1 & 0.8 & 0.2 & 0.2 & 0.3 & 0.0 & 0.0\\
0.5 & 0.3 & 0.1 & 0.6 & 0.4 & 0.1 & 0.6 & 0.4 & 0.1 & 0.1 & 0.0 & 0.0\\
0.3 & 0.7 & 0.3 & 0.3 & 0.7 & 0.3 & 0.2 & 0.7 & 0.3 & 0.1 & 0.2 & 0.1\\
0.1 & 0.1 & 0.1 & 0.1 & 0.2 & 0.1 & 0.2 & 0.1 & 0.1 & 0.1 & 0.3 & 0.3\\
0.2 & 0.5 & 0.2 & 0.1 & 0.4 & 0.1 & 0.2 & 0.5 & 0.1 & 0.3 & 0.7 & 0.3\\
0.1 & 0.4 & 0.4 & 0.1 & 0.4 & 0.4 & 0.1 & 0.5 & 0.5 & 0.2 & 0.4 & 0.5\\
0.5 & 0.2 & 0.0 & 0.4 & 0.3 & 0.0 & 0.4 & 0.3 & 0.0 & 1.0 & 0.1 & 0.0
\end{pmatrix}
\mathrm{B}=\begin{pmatrix}
0.98\\ 0.36\\ 0.63\\ 1.00\\ 0.40\\ 0.60\\ 0.10\\ 0.59\\ 0.41\\ 1.00
\end{pmatrix}
$$

$$
X_{\max(1)}=\begin{pmatrix}
[0.40,0.40]\\ [0.36,0.36]\\ [0.00,1.00]\\ [0.00,0.36]\\ [0.00,1.00]\\ [0.00,0.36]\\ [0.00,1.00]\\ [0.00,0.36]\\ [0.41,0.41]\\ [1.00,1.00]\\ [0.00,0.10]\\ [0.00,0.10]
\end{pmatrix}
\quad
X_{\max(2)}=\begin{pmatrix}
[0.40,0.40]\\ [0.00,0.36]\\ [0.00,1.00]\\ [0.36,0.36]\\ [0.00,1.00]\\ [0.00,0.36]\\ [0.00,1.00]\\ [0.00,0.36]\\ [0.41,0.41]\\ [1.00,1.00\\ [0.00,0.10]\\ [0.00,0.10]
\end{pmatrix}
\quad
X_{\max(3)}=\begin{pmatrix}
[0.40,0.40]\\ [0.00,0.36]\\ [0.00,1.00]\\ [0.00,0.36]\\ [0.00,1.00]\\ [0.36,0.36]\\ [0.00,1.00]\\ [0.00,0.36]\\ [0.41,0.41]\\ [1.00,1.00]\\ [0.00,0.10]\\ [0.00,0.10]
\end{pmatrix}
\quad
X_{\max(4)}=\begin{pmatrix}
[0.40,0.40]\\ [0.00,0.36]\\ [0.00,1.00]\\ [0.36,0.36]\\ [0.00,1.00]\\ [0.00,0.36]\\ [0.00,1.00]\\ [0.36,0.36]\\ [0.41,0.41]\\ [1.00,1.00]\\ [0.00,0.10]\\ [0.00,0.10]
\end{pmatrix}
$$

$$
XMean_1=\begin{pmatrix}
0.40\\ 0.36\\ 0.50\\ 0.18\\ 0.50\\ 0.18\\ 0.50\\ 0.18\\ 0.41\\ 1.00\\ 0.05\\ 0.05
\end{pmatrix}
\quad
XMean_2=\begin{pmatrix}
0.40\\ 0.18\\ 0.50\\ 0.36\\ 0.50\\ 0.18\\ 0.50\\ 0.18\\ 0.41\\ 1.00\\ 0.05\\ 0.05
\end{pmatrix}
\quad
XMean_3=\begin{pmatrix}
0.40\\ 0.18\\ 0.50\\ 0.18\\ 0.50\\ 0.36\\ 0.50\\ 0.18\\ 0.18\\ 1.00\\ 0.05\\ 0.05
\end{pmatrix}
\quad
XMean_4=\begin{pmatrix}
0.40\\ 0.18\\ 0.05\\ 0.36\\ 0.50\\ 0.18\\ 0.50\\ 0.36\\ 0.41\\ 1.00\\ 0.05\\ 0.05
\end{pmatrix}
$$

.

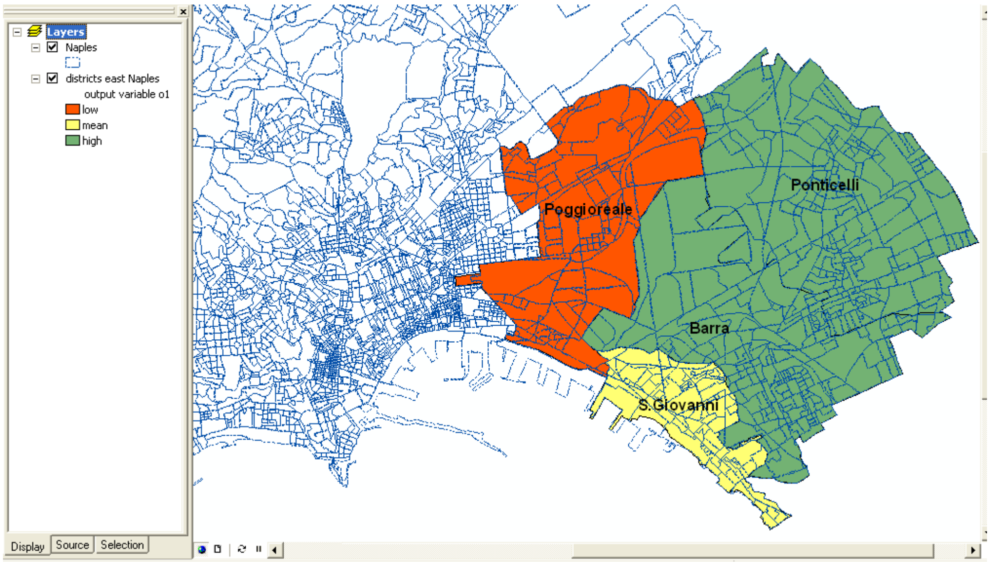
**Tab. 8 – Final linguistic labels for the output variables in the district “Barra”**

| Output variable | Linguistic label associated to $XMean_1$ | Linguistic label associated to $XMean_2$ | Linguistic label associated to $XMean_3$ | Linguistic label associated to $XMean_4$ |
|---|---|---|---|---|
| $o_1$ | high | high | high | High |
| $o_2$ | mean | mean | mean | mean |
| $o_3$ | low | low | low | low |
| $o_4$ | low | low | low | low |

## 5. Thematic maps and conclusions

Then we obtain four final thematic maps shown in Figs. 4, 5, 6, 7 for the output variable $o_1$, $o_2$, $o_3$, $o_4$, respectively. The results show that there was no housing development in the four districts in the last 10 years, there is difficulty in finding job positions and the remaining outputs $o_1$ and $o_3$ remain high in the indicated subzones.
In Fig. 8 we show the histogram of the reliability index Rel(O) for each subzone, where $O=[o_1,o_2,o_3,o_4]$.

**Fig. 4 – Thematic map for output variable o1 (Economic prosperity)**




.

**Fig. 5 – Thematic map of the output variable o2 (Transition into the job)**

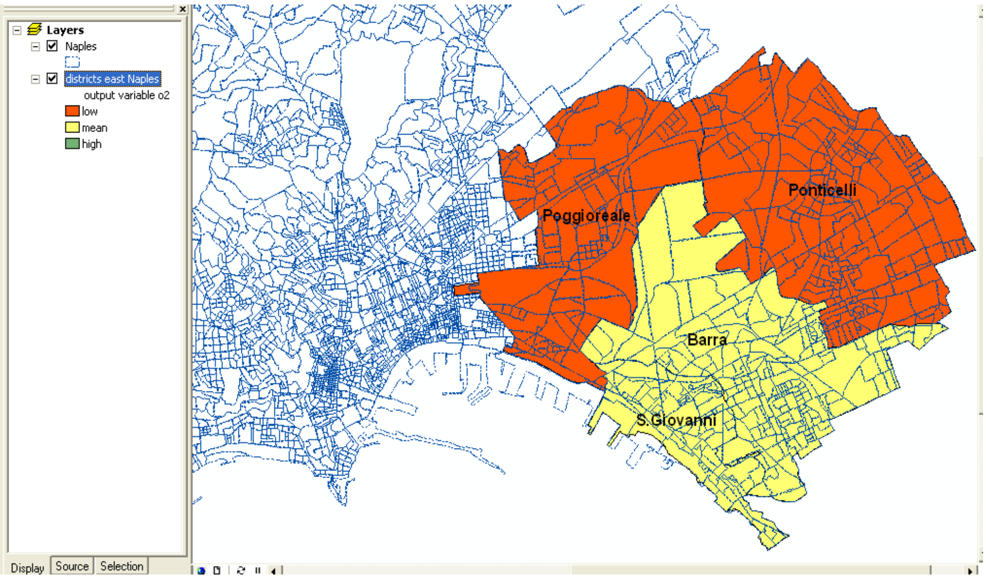



**Fig. 6 – Thematic map for the output variable o3 (Social Environment)**

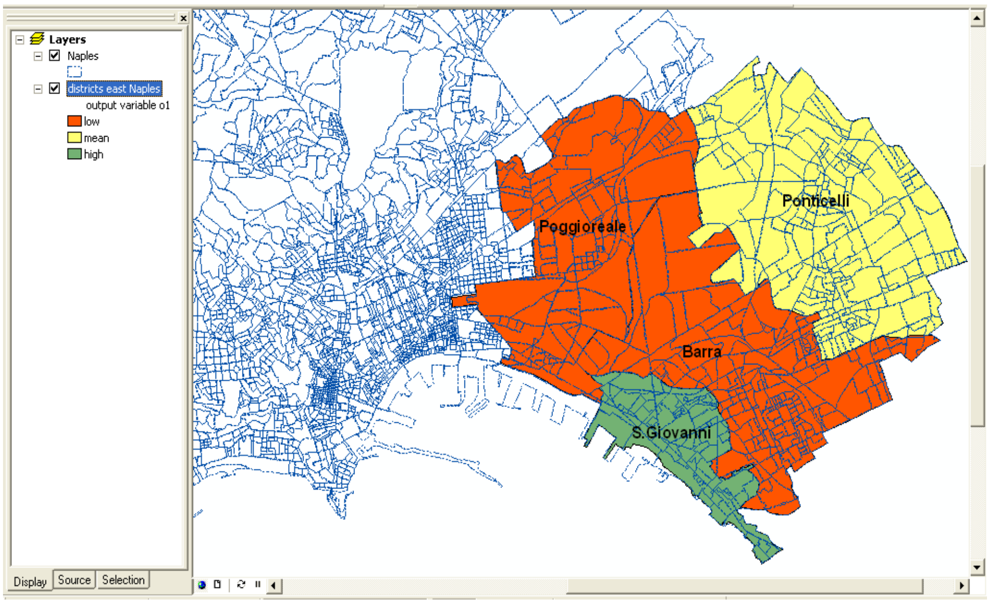


**Fig. 7 – Thematic map for the output variable o4 (Housing development)**

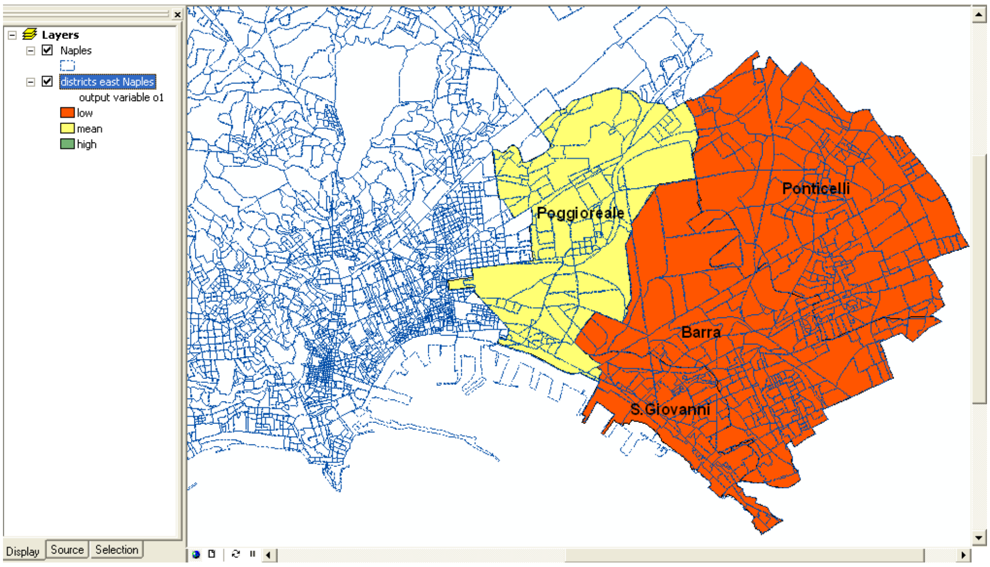
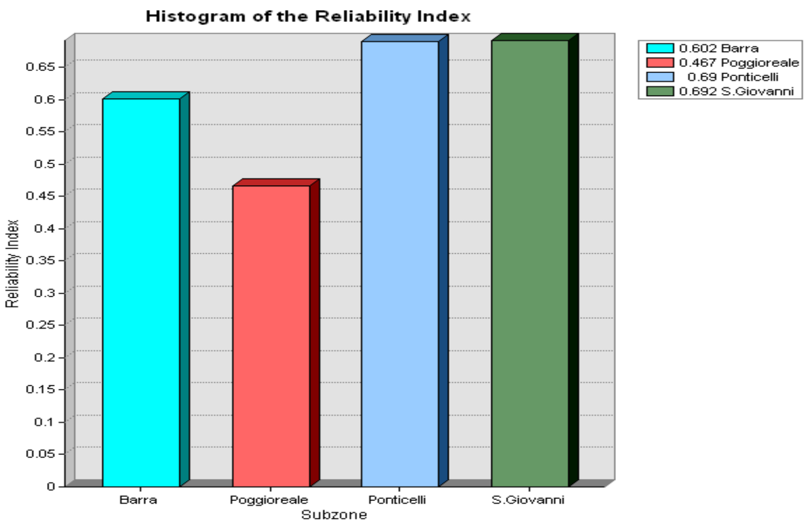



**Fig. 8 – Histogram of the reliability index Rel(O) for the four subzones**

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