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Editorial correspondence
Laboratory of Land Use Mobility and Environment
DICEA - Department of Civil, Architectural and Environmental Engineering
University of Naples "Federico II"
Piazzale Tecchio, 80
80125 Naples
web: www.tema.unina.it
e-mail: redazione.tema@unina.it

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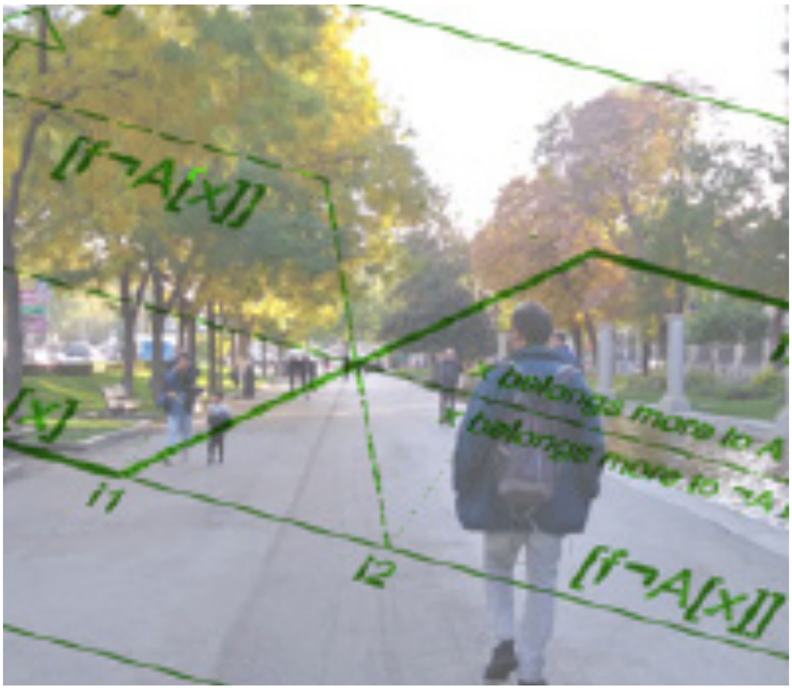

A METHODOLOGY FOR URBAN SUSTAINABILITY INDICATOR DESIGN

RICARDO ALVIRA BAEZA

Universidad de Murcia (Spain)
e-mail: ricardo.alvira@gmail.com
URL: https://www.um.es

ABSTRACT

In recent times we have witnessed proliferation of indicators and models for measuring sustainability. This reveals both the importance of the issue and the lack of common and shared scientific paradigm/ framework.
With the aim of advancing towards such common framework which enables quantitatively assessing the sustainability of our cities and societies, in this article it is explained a formal methodology for designing urban sustainability indicators based on Fuzzy Sets Theory. The interest of this methodology is threefold:

- Firstly, formal procedures enable testing, a most fundamental issue forgotten in many current proposals of sustainability indicators.
- Secondly, a formal procedure can become a common language allowing shared use of the indicators and facilitating their continuous improvement.
- And thirdly, fuzzy logic is widely used in computing and artificial intelligence, thus facilitating progressive automation of our sustainability monitoring models.

To help understand the procedure, the design of two indicators is reviewed, showing the applicability and easiness of the methodology.
Therefore, herein proposed methodology stands as an easy procedure, which generalization could allow us to increase the accuracy [testability] and shared used [efficiency] of our scientific research in sustainability as well as integrating it into artificial intelligence systems, increasing our capacity of successfully confronting current extremely high unsustainability of our society.

KEYWORDS:
Sustainability Measurement; Urban Sustainability; Indicator Design; Climate Change

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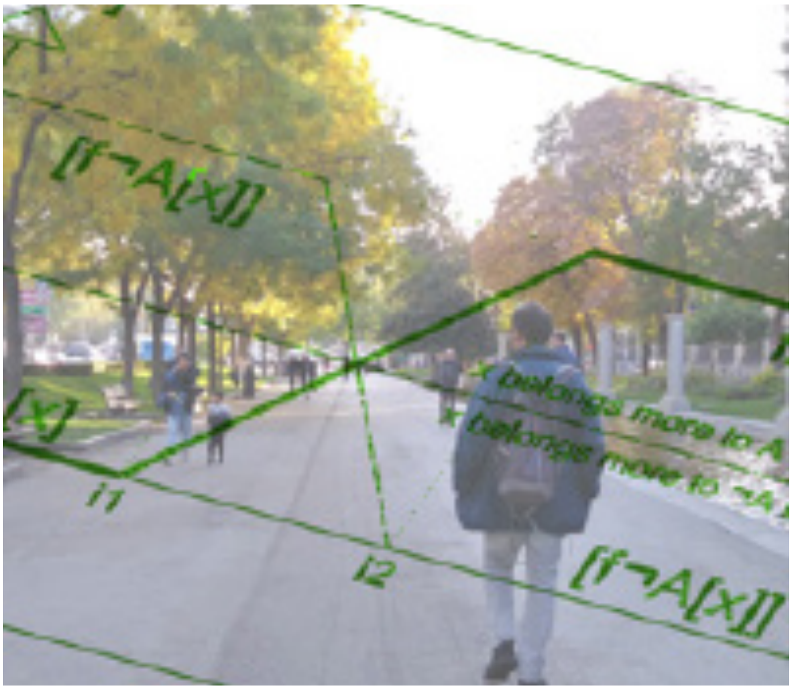

城市可持续性指标设计方法论

RICARDO ALVIRA BAEZA

Universidad de Murcia (Spain)
e-mail: ricardo.alvira@gmail.com
URL: https://www.um.es

摘要

最近，我们目睹了衡量可持续性的指标和模式激增。这既表明了这一问题的重要性，也表明缺乏共同和共享的科学范式/框架。
为了实现城市和社会可持续性定量评估的共同框架，本文阐述了一种基于模糊集理论的城市可持续性指标设计的正规方法。该方法的优点有三方面：
- 第一，正规程序能够允许进行测试，这是目前许多可持续性指标提案中常常容易忘记的一个最根本的问题。
- 第二，正规程序可以成为一种共同的语言，允许共享使用指标并促进其不断改进。
- 第三，模糊逻辑在计算领域和人工智能中得到了广泛的应用，从而促进了可持续发展监测模型的逐步自动化。

为了帮助明白该程序，对两个指标的设计进行了回顾，并说明了该方法的适用性和易用性。
因此，本文所提议的方法是一个简单的程序，其泛化性可以使我们提高可持续性科学研究的准确性[可测试性]和共享使用[效率]，并将其整合到人工智能系统中，从而提高我们成功应对当今社会极高不可持续性的能力。

关键词：
可持续性度量；城市可持续性；指标设计；气候变化

1 INTRODUCTION

Our current concern in relation to the increasing unsustainability of our society and development model, in conjunction with the increasing importance of cities to define such unsustainability, has taken to an everyday increasing number of different proposals for modeling and measuring urban sustainability. These proposals take the form of sustainability indexes or sustainability indicators dashboards, and their importance is that sustainability measurement stands as prerequisite for being able to increase it.

This constant increase of indicators and indexes for quantitatively assessing sustainability can be understood as something positive, as it increases the number of available tools for helping us moving towards sustainability. But it also conceals two negative issues:

- the lack of a common framework for sustainability measurement[1] leads to every new proposal defining its own framework, which often cannot be linked to most existing knowledge. This implies great effort and forgets two important issues: connection to previous proposals makes scientific research more efficient and usually enables its easier testing, the last being an often forgotten yet fundamental issue for science: an untested proposal is unscientific by definition;
- contradiction between statements made by different models generates lack of consensus greatly hindering making the required decisions for advancing towards sustainability. Most of these decisions are collective decisions; i.e., decisions that need to be made by consensus among many agents with different preferences/interests. Which model should we use then if different models suggest different courses of action that imply different utility for different agents?

Advancing towards shared/consensual knowledge in Sustainability currently stands as prerequisite for advancing towards Sustainability. With this goal, in this article a methodology for sustainability indicator design is explained that allows us to understand what these indicators should measure and how, aiming to set a common framework that enables their shared used by the scientific community.

To define this framework, a review of Sustainability conceptualization is undertaken from the two approaches to *logic* from Set or Class Theory[2]:

- Classic Set Theory or Boolean Logic (Boole, 1854; Hacking, 1995) allows us to conceptualize the class of sustainable Cities [S] as opposed or complement to that of Unsustainable Cities [¬S];
- Fuzzy Sets Theory or Fuzzy Logic (Zadeh, 1965) allows us to conceptualize the sustainability degree of a city as its Grade of membership to the set or class of sustainable cities [S].

The second approach is better fitted to our objectives; therefore, we build the methodology for designing the indicators on Fuzzy Logic/Fuzzy Sets Theory. For greater clarity, two urban indicators are reviewed using herein proposed methodology. Prior to the review, it is convenient to state two easy definitions of sustainable city built on two perspectives:

- from a probabilistic perspective, a sustainable city is that maximizing its probability of indefinitely enduring;
- from an optimality perspective, a sustainable city is that maximizing the degree to which it is in its optimal state[3].

Let us start by reviewing the conceptualization of sustainability according to Classical Sets Theory.

---

[1] Beware by common framework we do not refer to a unique context-independent model to be used anywhere around the world, but to the logical framework underlying the models. Different contexts may imply the relevant variables and indicators for sustainability (their sustainability thresholds) are different.

[2] There is a difference between a set and a class (i.e., a set is a class that belongs to another class) yet for the present work both terms are considered to be synonym and equivalent to class.

[3] Although this definition is somewhat redundant, it could be more briefly stated as "a city which is in its optimal state" (Alvira, 2017), it help us to easier understand herein explained approach.

2 CLASSIC SET THEORY OR BOOLEAN LOGIC: SUSTAINABILITY AND UNSUSTAINABILITY AS COMPLEMENTARY SETS[4]

Classic Set Theory *groups* objects into different classes by assigning each object to each set or class by a binary membership function. Given an object x and a set or class A, a value *zero* means that x does not belong to A (therefore, it belongs to ¬A), and a value *one* means that x belongs to A.

$$f_A[x] = \begin{matrix} 0\ if\ x\ \notin a \\ 1\ if\ x\ \in a \end{matrix}$$

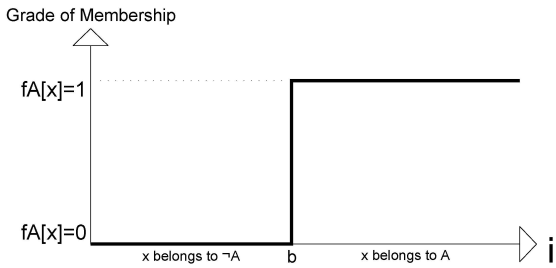

Fig.1 Binary Membership function, where b is the value of some variabl i describing x, which separates null from full membership of x to class A

Membership as conceptualized by Classic Set theory or Boolean logic implies therefore the idea of mutually exclusive classes or concepts that can be defined as those whose intersection is empty and their union provides the universe of discourse:

$$A \cup \neg A = \Omega\ [R] \tag{1}$$

$$A \cap \neg A = \emptyset \tag{2}$$

This last statement expresses the *Duality Law* (Boole, 1854) as a condition for the interpretability of logical functions, which is a formalization of *Aristotle's Non-contradiction Principle*. It is possible building a first conceptualization of Urban Sustainability on above statement. If we consider the set that includes all cities and we divide it into two subsets:

- we designate S or *Sustainability* the set composed by all sustainable cities;
- we designate ¬S or *Unsustainability* the set composed by all non-sustainable cities.

Following above criteria the union of S and ¬S (sustainable and non-sustainable cities) must contain all cities, while their intersection must be empty:

$$\begin{matrix} S \cup \neg S = 'Cities' = \Omega\ [R] \\ S \cap \neg S = \emptyset \end{matrix} \tag{3}$$

We can represent it as:

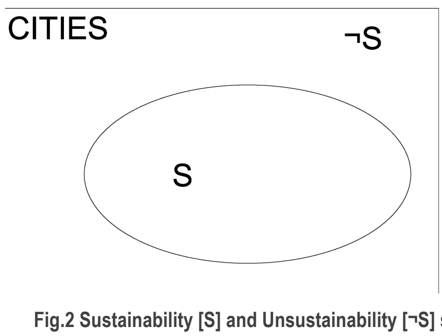

Fig.2 Sustainability [S] and Unsustainability [¬S] sets are complement in the universe Cities

[4] This chapter and the following are a reformulation and update of Alvira (2018 [2013])

The drawback of this approach from Boolean logic is that though being theoretically correct (in the long run a binary approach is the only possible; a city endures or not) it does not fit above proposed definitions:

- a city may have a *continuum* range of probabilities to indefinitely endure;
- a city may be *closer or further* from its optimal state.

Being able to characterize cities consistently with above two definitions requires confronting it from Fuzzy Sets Theory or Fuzzy Logic.

3 FUZZY SET THEORY AND LOGIC: SUSTAINABILITY DEGREE AS GRADE OF MEMBERSHIP

A Fuzzy set is a class characterized by a membership function $f_A[x]$ that associates to each element x of a universe X a number in the range [0,1]; i.e., a class with a continuum of grades of membership[5]:

$$a = \{[x, f_A[x]] | x \in X\}$$
$$f_A[x] \rightarrow [0,1] \quad (4)$$

Fuzzy Logic is a development of Boolean logic to confront intermediate situations that allow *grades of membership and exclusion*; widening the applicability of the Non-contradiction Principle.

While classical logic can only be used with mutually exclusive concepts (i.e., concepts that must be true or false applied to an object) *fuzzy logic can be also used with any concept or quality that can be partly true. Any object can be characterized by the degree it possess some quality and the non-quality*; i.e., by the degree it belongs – its grade of membership – to a class and to its opposite or complement.

A fuzzy membership function can take any value in the range [0-1], which allows us to measure urban sustainability and unsustainability in terms of *sustainability / unsustainability degree:*

- the *Sustainability Degree* of a city I at a moment T is its *grade of membership* to S and we designate it as $S_T[I]$

$$S_T[I] = f_s[I] \quad (5)$$

- the *Unsustainability Degree* of a city I at a moment T is its *grade of membership* to ¬S and we designate it as $\neg S_T[I]$

$$\neg S_T[I] = f_{\neg s}[I] \quad (6)$$

Therefore, the Sustainability Degree of a city I at any moment T has a value in the range 0 and 1, and we can assign different meaning to said value:

- $S_T[I] = 1$ the membership to *Sustainability* class is complete, and therefore the grade of membership to *Unsustainability* class is zero;
- $0 < S_T[I] < 1$ the city has a grade of membership to *Sustainability* class, complementary to its grade of membership to *Unsustainability* class;
- $S_T[I] = 0$ the grade of membership to *Sustainability* class is zero, and therefore the membership to *Unsustainability* class is complete.

We see Fuzzy Sets Theory allows us to characterize urban sustainability consistently with above definitions. Let us then review some properties of the fuzzy sets which are useful for understanding herein proposed methodology.

---

[5] This definition and the majority that follow are from Zadeh (1965).

3.1 PROPERTIES OF FUZZY SETS

Fuzzy sets have four properties interesting for our proposal:

- Complementary set or complement, the complement of a set A is denoted as ¬A and defined as:

$$f_A[x] = 1 - f_{\neg A}[x] \tag{7}$$

- Containment, if A is contained in B its membership function $f_A[x]$ is smaller than B $f_B[x]$ for any x:

$$\forall x \in X: A \subset B \rightarrow f_A[x] \leq f_B[x] \tag{8}$$

This property has great relevance for urban sustainability analysis because it imposes an important condition to the Sustainability Degree of a city; it is equal or lower than the Sustainability Degree of the environment that contains it.

- Union, the union of two fuzzy sets A and B with respective membership functions $f_A[x]$ y $f_B[x]$ is a fuzzy set C, which membership function is $f_C[x]$

$$C = A \cup B \rightarrow \forall x \in X: f_C[x] = max\big[f_A[x] \cap f_B[x]\big] \tag{9}$$

- Intersection, the intersection of two fuzzy sets A and B with respective membership functions $f_A[x]$ y $f_B[x]$ is a fuzzy set C which membership function is $f_C[x]$:

$$C = A \cap B \rightarrow \forall x \in X: f_c[x] = min\big[f_A[x] \cap f_B[x]\big] \tag{10}$$

To summarize, above formulas allow us to relate membership functions to Sustainability and Unsustainability classes as:

$$f_S[I] + f_{\neg S}[I] = 1 \tag{11}$$

Therefore, the *Sustainability Degree* and the *Unsustainability Degree* of a city are linked by the equation:

$$S_T[I] = 1 - \neg S_T[I] \tag{12}$$

Above equation means that any *lack of complete Sustainability necessarily implies some unsustainability degree*, and $S_T[I] = 0,5$ becomes a *limiting* value that separates the cities that are more *sustainable* than *unsustainable* ($S_T[I] > 0,5$) from the cities that are more *unsustainable* than *sustainable* ($S_T[I] < 0,5$).

$$S_T[I] > 0,5 \leftrightarrow S_T[I] > \neg S_T[I] \tag{13}$$

$$S_T[I] < 0,5 \leftrightarrow S_T[I] < \neg S_T[I] \tag{14}$$

After reviewing these basic properties of fuzzy sets, we review below a useful tool for working with fuzzy membership functions: their graphic representation.

3.2 GRAPHIC REPRESENTATION OF MEMBERSHIP FUNCTIONS

Graphic representation of membership functions is always advisable since it provides a lot of information that is not always easily noticeable in the mathematical formulations. Additionally, it allows us to understand some important issues: the first is that if we consider a membership function on a continuous variable i that defines the grade of membership of an element x to a class A, graphical representation allows us to see the existence of two especially relevant values or points:

- A value i_1 so that if $I \leq i_1$ then x membership to class A is zero (and therefore, its membership to class ¬A is complete)

$$\exists i_1: i \leq i_1 \leftrightarrow f_A[x] = 0 \wedge f_{\neg A}[x] = 1 \tag{15}$$

- A value i_2 so that if $I \geq i_2$ then x membership to class A is complete (and therefore, its membership to class ¬A is zero)

$$\exists i_2: i \geq i_2 \leftrightarrow f_A[x] = 1 \wedge f_{\neg A}[x] = 0 \tag{16}$$

Both values are fundamental for the design of an urban sustainability indicator in relation to some variable information i of a city I. *We designate* $\boldsymbol{i_1}$ *as its unsustainability limit or threshold, and* i_2 *as its sustainability limit or goal.*

$$\boldsymbol{f_A[x] = max\left[min\left[\frac{i - i_1}{i_2 - i_1}, 1\right], 0\right]}$$

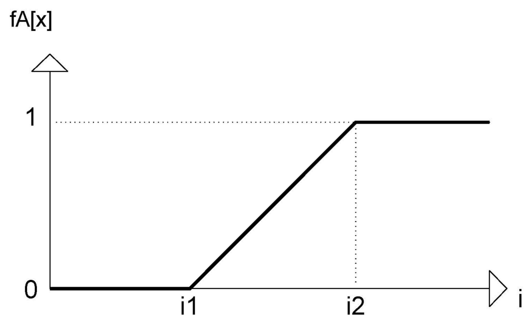

Fig.3 Linear fuzzy membership function, i_1 is the value of i for which x membership to A becomes 0 and i_2 is the value of i for which x membership to A becomes complete

The necessary existence of these limits allows us to define a *relevant variable for urban sustainability* as a variable for which at least one unsustainability limit and one sustainability limit exist [they may be or may not be known]. As consequence, the sustainability limits of a variable are the delimiting values for the range of the variable i that produces fuzzy membership of I to S; i.e., the extreme values of the range which imply either complete membership of the city to S or to ¬S classes.

The second interesting issue of graphical representation is that it allows synthesizing the membership to a set and to its complement in one graphic:

$$\boldsymbol{f_A[x] = max\left[min\left[\frac{i - i_1}{i_2 - i_1}, 1\right], 0\right]}$$

$$\boldsymbol{f_{\neg A}[x] = 1 - f_A[x]}$$

$$\boldsymbol{f_{\neg A}[x] = max\left[min\left[1 - \frac{i - i_1}{i_2 - i_1}, 1\right], 0\right]}$$

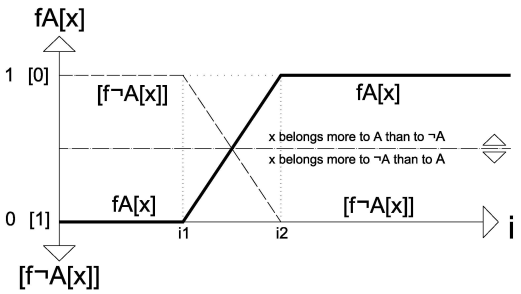

Fig.4 Membership function of an element x to A and ¬*A* sets. There is a horizontal symmetry at $f_A[x] = 0.5$, which separates the values of i for which x belongs more to A of the values of i for which x belongs more to ¬*A*

And this property implies that membership to S and ¬S can be represented in the same graphic, but even if we represent only one of them (it is usually more interesting representing membership to S) then membership to the complement (i.e., membership to ¬S) is easily obtained.

4 DESIGNING SUSTAINABILITY INDICATORS: SUSTAINABILITY DEGREE OF A CITY IN RELATION TO THE VARIABLES THAT DESCRIBE IT

We have conceptualized the sustainability degree of a city I as its grade of membership to class S, but it is necessary to state that it depends on many different variables and relationships between variables and usually we are not able to calculate it with only one formulation. Thus, we approach the modelization progressively. We analyze the concept *Sustainable* to detect the concepts or qualities S_i that we expect in a sustainable city (i.e., that we expect to be *true* when referred to a *sustainable city)* and we review the information that defines the truth value[6] of these concepts or *propositions* when referred to the city.

For instance; we usually state that a sustainable city must have *high employment levels*; *accessible public transport service*; *adequate provision of green areas*, etc... And indicators measure the degree of truth of those propositions referred to the city (i.e., the degree of truth of the statements 'city I has *high* employment levels'; 'city I has accessible transport',...); which can be modeled as *membership functions* to those different classes implied by said propositions (to the class of the cities with high employment levels, to the class of the cities with accessible public transport service…).

Urban sustainability indicators are equivalent to membership functions of the city to the different classes S_i *contained* in class S for each possible range of different relevant variables i, and its maximum and minimum values have the following meanings:

- $S[I_i] = 0$ means *null membership* to S_i (and complete membership to $\neg S_i$); the city does not have at all a quality expected in a *Sustainable City*;
- $S[I_i] = 1$ means *complete membership* to S_i (and null membership to $\neg S_i$); the quality expected in a *Sustainable City* is completely present in the city.

Therefore, the unsustainability/sustainability *limits* of the relevant variables for each class S_i are the values i_1 and i_2 at which null or complete membership to classes S_i and $\neg S_i$ are reached. Both values are especially relevant for indicators formulation, which we review below.

4.1 SUSTAINABILITY AND UNSUSTAINABILITY LIMITS

A variable i is relevant for the sustainability of a system I if and only if different values of the variable can imply a variation on both city sustainability and unsustainability, being the sustainability and unsustainability limits, those values of the variable for which the city reaches its maximum possible membership to classes S and ¬S[7].

These limits may or may not be known, but in general, the formalization of indicators can only be done if we are able to establish (even if approximately) their value.

In their more *simple* form, the limits are two parameters that divide in three different zones the impact on the *Grade of membership* of a city I to any class S_i implied in S, for the range of possible values of i:

- the first is value of i for which I reaches null membership to S_i which we designate as *Unsustainability limit or threshold;*
- the second is value of i for which I reaches complete membership to S_i which we have designated as *Sustainability limit or goal*.

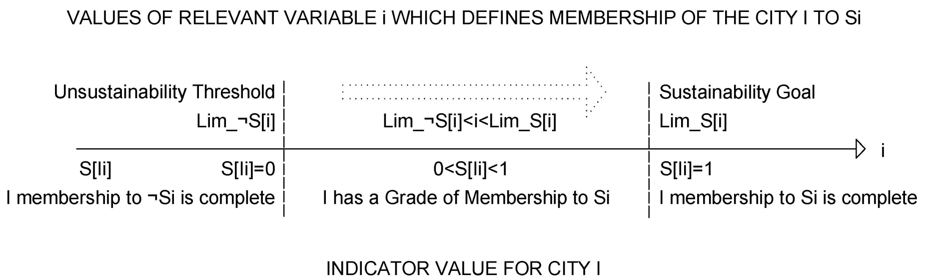

Fig.5 Relation between i values, thresholds and sustainability degree

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6 The concept of Truth Value (Fuzzy Logic) is equivalent to the concept of Grade of Membership (Fuzzy Sets Theory).

7 The majority of relevant variables do not imply complete membership of the city to classes S and ¬S, but their complete membership to classes Si and ¬Si. Therefore the worst value of the variable does not indicate complete membership to ¬S, but the maximum membership to ¬S such variable can imply. Also, the optimal value of the variable does not imply complete membership to S but the maximum membership to S said variable can imply.

Additionally, there are some important issues that need to be highlighted regarding the limits:

- they can be *exact* values but also *ranges of values* or even *dynamic values*[8]: the *state* of the system may modify the value of the limits and any *change* in the system–environment (including evolutionary processes) may change the limits;
- containment property implies that sustainability degree of any city is always equal or lower than that of its environment; which may impose *additional limits;*
- for some variables urban sustainability may imply *more than two limits;*
- *different contexts* may imply *different limits.*

4.2 FORMULATION OF SUSTAINABILITY INDICATORS FOR DIFFERENT TYPES OF VARIABLES

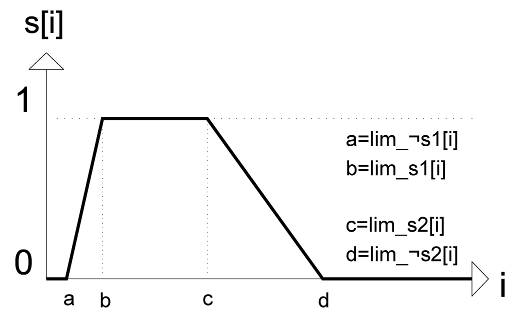
We have thus arrived to a conceptualization of a sustainability indicator as a membership function of a city I to a subclass S_i (partly) contained in class S regarding the possible values of some relevant variable information 'i'. And 'i' sustainability/unsustainability limits are fundamental for indicator formulation. Though there are many different possible formulations (linear, quadratic, logarithmic ...), in most cases, a linear function can sufficiently accurately model membership[9]. These linear functions can be formalized building on a four limits (two sustainability and two unsustainability limits) formulation:

Indicator Formalization	Graphic Representation

$$s[i] = max\left[min\left[\frac{i-a}{b-a}; 1; \frac{d-i}{d-c}\right], 0\right]$$

Thus:

$$S[I_i] = max\left[min\left[\frac{i - \underset{\neg s1}{lim}[I_i]}{\underset{s1}{lim}[I_i] - \underset{\neg s1}{lim}[I_i]}; 1; \frac{\underset{\neg s2}{lim}[I_i] - i}{\underset{\neg s2}{lim}[I_i] - \underset{s2}{lim}[I_i]}\right]; 0\right]$$



Source: own elaboration using the following codes:

1) $S[I_i]$ _ Value of the sustainability indicator I_i for a system I
2) i_ value of the relevant variable (it can be an aggregation of variables)
3) $Lim_{\neg s1}[Ii]$_unsustainability threshold 1 for the system I related to variable i.
4) $Lim_{\neg s2}[Ii]$_ unsustainability threshold 2 for the system I related to variable i.
5) $Lim_{s1}[Ii]$_ sustainability limit or goal 1 for the system I related to variable i.
6) $Lim_{s2}[Ii]$_ sustainability limit or goal 2 for the system I related to variable i.

Tab.1 Membership Function and Graphic Representation of a four limits variable i

Starting from above function, many different types of indicators can be built; using two or three limits; substituting some or all linear functions by non- linear functions (Alvira, 2017a, 2017b, 2018). Since our aim in this article is to explain how this approach can be used to easily design sustainability indicators, instead of an extensive review of possible functions, we focus in reviewing formulation of two indicators using herein explained methodology.

4.3 EXAMPLES OF INDICATORS DESIGNED USING THE PROPOSED APPROACH

To better understand the proposed methodology, below two indicators are explained whose formulation implies different level of difficulty:

[8] For an explanation related to the limits of global ecosystem 'Earth' refer to Steffen et al. (2015) who suggest that if certain variables of a system get close to their unsustainability thresholds, the sustainable range of values for other relevant variables changes.

[9] In my opinion, unless an appreciable accuracy increase is achieved, it is not convenient to use more complicated functions, since it may hinder the comprehension of indicators and as consequence their shared use.

- firstly, we review the formalization of an indicator to assess the *optimality of the Green Areas Provision of a city*. It is an easy formulation to assess an issue about which there is little controversy at present;
- Secondly, we review the formalization of an indicator to assess the degree to which *Population Density* places an urban area between its optimal and worst possible states. It is necessary to use a somewhat more complex formulation (it requires four limits), and it is also necessary to estimate two unsustainability thresholds since we find scarce or no proposals.

Let us review these indicators[10].

Indicator to assess the sustainability of green areas provision [GA]

Sources and related indicators:

- Hernández Aja et al., 1997;
- AEUB, 2010. Indicator 25. Green Areas Provision per inhabitant;
- JSBC, 2011. Indicator 2.1.2. Adequate provision of parks and open spaces;
- MFOM, 2012. Indicators EVB.05.23 & EVB.05.26. Green Areas provision (New Developments & Existing Urban Areas);
- Alvira, 2017a. Indicator Q3.1. Green Areas Provision and Functionality.

Indicator description, sustainability limits and calculation.

It is a relatively easy to formulate indicator, for an issue on which there is enough agreement among experts: *what is the per capita surface of green areas that approaches a city to its optimal state.* There is wide agreement on the importance of urban green areas to define the quality of life of the population and urban sustainability[11], which is sustained on several perspectives:

- their use as a leisure, walking and sports space (AEUB, 2010; MFOM, 2012);
- their nature of 'social relation' space accessible to the entire population, which makes them spaces that promote social cohesion (Hernandez Aja et al., 1997; Higueras, 2009);
- they can be designed as 'green infrastructure', providing increased climate change adaptation (Beauchamp & Adamowski, 2013; Salata & Yiannakou, 2016; TCPA/The Wildlife Trusts, 2012; Zucaro & Morosini, 2018);
- they have psychological benefits by enabling people's contact with nature (Prescott-Allen, 2001).

There is also high agreement that the optimal provision of green areas is between 10 and 15 sq.m per resident/inhabitant, finding more or less compatible proposals from different authors:

- Hernández Aja et al. (1997) proposes different provision ratios for different types of urban fabric and green areas. At the overall city level the author proposes: proximity Parks [several types and surfaces] = 8 sq.m/inhabitant; city Scale Parks [Urban Parks]: 5 sq.m/inhabitant; city total provision: 13 sq.m/inhabitant.
- JSBC (2011) proposes an *acceptable* value of Green Areas provision of 7 sq.m/inhabitant and an optimal value of 13 sq.m/inhabitant;
- WHO (quoted by several authors) suggests between 10 y 15 sq.m/inhabitant;
- AEUB (2010) proposes a 10 sq.m/inhabitant minimum and a desirable goal of 15 sq.m/inhabitant;
- MFOM (2012) proposes between 10 and 12 sq.m/inhabitant of Green Areas for both new urban developments and as overall city wide provision. However, for urban areas within existing cities the authors suggest a 15 sq.m/inhabitant optimum provision.

[10] Noteworthy, the indicators we review below are proposed for neighborhood type areas in developed countries cities. Other contexts could require different designs.

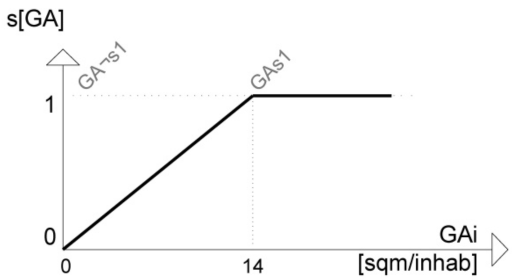
[11] "Green spaces are considered by the World Health Organization [WHO] 'essential' spaces for the benefits they bring in the physical and emotional well-being of people and for helping to mitigate the urban deterioration of the city, making it more livable and healthy' (AEUB, 2010)

We observe high similarity in the proposals, with a 13-15 sq.m/inhabitant range as optimum value (slightly lower values are only proposed in central areas of the city where clogging prevents reaching higher values), being possible to adopt for the indicator the middle value of said range: 14 sq.m/inhabitant.

Therefore, the two sustainability/unsustainability limits for the indicator are:

- sustainability goal GA_{s1}: 14 sq.m/inhabitant;
- unsustainability threshold GA_{is1}: 0 sq.m/inhabitant.

The graph and formula of the sustainability function are:

Graphic Representation	Indicator Formalization
	$s[GA] = \max\left[\min\left[\frac{GA_i - GA_{\neg s1}}{GA_{s1} - GA_{\neg s1}};1\right];0\right]$ Being: GA_ Green Areas provision Indicator; GA_i_ Green Areas provision in the assessed Area GA_{s1}_ Sustainability goal in Green Areas $GA_{\neg s1}$_Unsustainability threshold in Green Areas provision.

Tab. 2 Indicator for measuring the sustainability of Green Areas Provision

Since the unsustainability limit $GA_{\neg s1}$ is 0 sq.m/inhabitant; the sustainability limit GA_{s1} is 14 sq.m/inhabitant, and the relevant variable GA_i (Green Area sq.m per inhabitant) cannot have a lower value than 0, it is possible to simplify above function as:

$$s[GA] = \min\left[\frac{GA_i}{14};1\right] \quad (1)$$

This function is similar to many of the usual indicator formulations, which allows us to understand why it is sometimes possible to *intuitively* confront indicators design achieving coherent results[12].

<u>Further comments</u>

It is worth highlighting that as important as assessing the per capita surface of Green Areas are some issues which have not been included in the indicator to avoid complicating the explanation:

The first issue is *Green Areas quality/functionality* (WHO, 2016) which covers aspects such as: the percentage of landscaped area and type of landscaping, equipment, lighting, perceived and real safety, acceptable noise levels (especially in small surface GA). Some proposals to model it are:

- AEUB (2010), which proposes criteria differentiating two scales: neighborhood spaces: it suffices that 50% of the surface is permeable; urban parks: it is necessary to assess their Functionality, which is linked to a series of aspects that require individual modeling, and subsequent joint assessment[13];
- WHO (2016) suggests using the *Normalised Difference Vegetation Index* [NDVI], which it describes as 'an indicator of the degree to which an area is *green*'.

The second issue is *Green Areas accessibility*. In order to assess it several authors have proposed greater distances are acceptable the lower the expected frequency of use, setting thus different optimal distances according to green areas dimension/nature. For example, MFOM (2012) proposes the following maximum

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[12] However, later we review another more difficult indicator which cannot be *intuitively* designed, supporting the interest of herein proposed methodology.

[13] AEUB (2010), Indicator 28. Index of functionality of Urban Parks [Surface > 1Ha]. Although the goal of the indicator is assessing biodiversity, evaluated aspects are closely related to the design quality of the green areas. Positive aspects in the valuation of the parks are: Tree coverage in percentage; Shrub Coverage in percentage; Lawn coverage in percentage; Water coverage in percentage; Number of large trees; Number of trees of average size; Number of trees of small size; Diversity of tree and shrub species. Negative aspects in the valuation of the parks are: Artificial Surface in percentage and Distance to natural habitats.

distances from green areas to expected users: green areas up to 500 sq.m maximum distance 200 m; green areas up to 5,000 sq.m maximum distance 750 m; green areas up to 1 Ha maximum distance 2 km; green areas up to 10 Ha maximum distance 4 km. And the third issue is that the overall layout of green areas throughout the city should use their high capability for *climate change adaptation*. Green areas distribution should not only take into account human accessibility and biodiversity connection, but also maximizing heat island mitigation, flooding prevention and optimizing water management (Galderisi, 2014; Zucaro & Morosini, 2018)[14].

Indicator to assess the sustainability of population density

Source and related indicators:

- Jacobs, 1961;
- AEUB, 2010. Indicador 01. Population Density;
- MFOM, 2012. Existing fabrics. Indicator 01. Population Density;
- Alvira, 2017a. Indicador Q1.1. Population Density;
- USGBC, 2018. Compact Development.

Indicator description, sustainability limits and calculation

It is a somewhat more complicated indicator to formulate. Experts agree cities are unsustainable when their population density is very low, but also that they are unsustainable when their population density is very high. Thus, there is an intermediate range of density values which are the optimal/most sustainable states of urban areas (Fariña Tojo & Naredo, 2010; Güneralp et al., 2017; MFOM, 2012; Jacobs, 1961).

The characterization of such states requires using a formulation that incorporates four sustainability limits (two sustainability and two unsustainability limits). However, we find few proposals regarding which population density values most approach cities to their worst possible states, so deeper review is necessary in order to establish these values. For clarity, we first review which limits have been proposed as optimal population density situations. Most consistent proposals have been made by two authors: Jacobs (1961) reviewed the density parameters of several *high vitality and diversity neighborhoods* in US cities, finding they located in an average range of 90 and 185 housing/ha[15], usually considered as *high values*. If we assume an average occupation of 2.5 persons/housing, we obtain an optimum density range between 225 and 463 inhab / Ha[16]. From her study, Jacobs suggested that excessively low or high densities are negative for cities and their inhabitants, i.e., that *there is an optimum range of densities to achieve attractive environments, with vitality and diversity. Agencia de Ecología Urbana de Barcelona* (AEUB, 2010), broadened the previous approach, by relating the range of optimal densities to complementary issues:

- *very low densities* imply a dispersed city model that requires consuming a lot of resources[17] and makes contact and shared use of the city difficult (public facilities, public transport,...);

[14] While the resilience/sustainability of the city could be further increased by incorporating urban orchards into Green Areas (Bianconi et al., 2018), this should be assessed using other indicators which assess membership to other classes such as biocapacity use or social relation spaces provision.

[15] Average values of the lower and upper limits for New York, Boston, Philadelphia and San Francisco neighborhoods that Jacobs (1961) considers as having *high vitality and diversity*. Building on her review, Jacobs stated that the prevailing paradigm in the USA that linked high urban quality to urban sprawl was wrong.

[16] Jacobs (1961) suggested a minimum value of 100 housing/acre (approx. 250 housing/Ha) of net density, but she indicated that a density value may had different meanings in different environments. Thus, she suggested that the central areas of the cities that have been conformed over time, have greater age of buildings and a greater variety of typologies and uses, admit higher densities than residential areas built in reduced time intervals, which present great homogeneity. In this last case, high density may imply conflicts and uprooting.

[17] Moore (2011) finds direct relationship between density and urban metabolism; an increase in density of 40 people/sq.km implies a reduction of approximately 0.06 hag in the per capita ecological footprint of the urban area.

— *very high densities* imply excessive congestion, and can lead to indirect consumption increase[18] in the form of greater demand for travel or second residence (MMA, 2007).

AEUB (2010) suggested an optimal density range of 220-350 inhab/Ha. Subsequently, the authors somewhat extended the optimum density range to 200-400 inhab/Ha (MFOM, 2012)[19].

There is some similarity among the three ranges of values, the range 220-350 inhab/Ha (AEUB, 2010) standing as acceptable *sustainability limits* for a varied range of urban environments.

On the contrary, we have not found proposals to establish *unsustainability thresholds*, so a review from several approaches is undertaken below: the first approach is based on the comparison of the two optimal ranges proposed by AEUB (AEUB, 2010; MFOM, 2012). Assuming the range 220-350 inhab/Ha (AEUB, 2010) as *optimal range*, and values 200 and 400 inhab/Ha (MFOM, 2012) as *excellent values*, the later values should imply equal variation in the indicator value. Assuming then the minimum possible value (i.e, zero) as unsustainability threshold $DP_{\neg s1}$, then $DP_{\neg s2}$ can be calculated by means of proportionality rules:

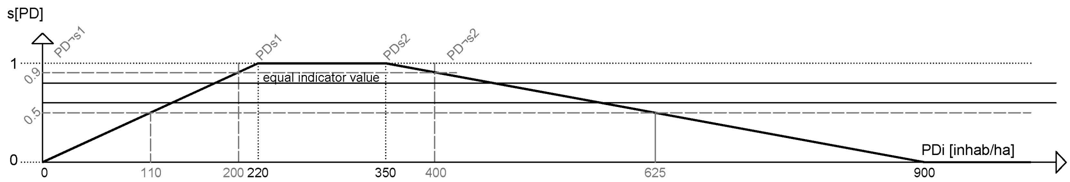


Fig.6 Unsustainability thresholds estimated by proportionality rules

Assuming 0 inhab/Ha as unsustainability threshold $DP_{\neg s1}$ and a linear function then 200 inhab/Ha provide value 0.9 for the indicator. Assigning the same indicator value to 400 inhab/Ha, 900 inhab/Ha value is obtained as unsustainability threshold $DP_{\neg s2}$.

The second approach is building on the concept of *ecological carrying capacity*. A reduced population density implies greater soil consumption to sustain the same population, reducing the area of bio-productive territory available to sustain said population. In Alvira (2017a) the available territory for urbanization in Spain is measured according to Ecological Footprint criteria, obtaining a maximum of 0.0715 hag-eq (447 sq.m) per capita assuming the current population is equally distributed and the available territory is used at 100% for residential use. This figure implies 22.4 inhab/Ha density. Applying maximum unsustainability criteria stated in said text, complete unsustainability is achieved if each inhabitant uses 1.7 times the maximum globally per capita available surface, i.e., when a person needs 0.128 hag-eq (800 sq.m) of urban territory equivalent to 12.5 inhab/Ha density. Since not all urban territory is residential, the previous figure is rounded up to $DP_{\neg s1}$=15 inhab/Ha. From said value $DP_{\neg s2}$ can be calculated by proportionality obtaining $DP_{\neg s2}$ = 862.5 inhab/Ha.

The third is reviewing the values proposed in different regulations:

— the maximum value of population density that we have found in Spanish legislation is in Canary Islands (CAC, 2017) where a maximum of 400 inhab/Ha gross density in residential areas is accepted, reaching a maximum of 500 inhab/Ha in urban centers rehabilitation;

[18] The graph that relates energy consumption to housing density is U-shaped. Consumption in environments with low housing density is very high (caused mainly by transportation and single-family housing), and decreases as density increases, then it stabilizes, yet from certain higher density values it increases again as people tend to make more trips for leisure and further away. This has been called 'substitution hypothesis'; when urban areas become excessively dense, their inhabitants experience a 'lack of space' that they seek to replace by undertaking more trips away from 'congestion' or having second homes in the countryside (SEI/TUB, 2010).

[19] In Spain, most dense cities are Barcelona and Bilbao (198 and 196 inhab/Ha) (OSE, 2008). In Madrid most dense neighborhoods have net population and houses densities around 700 inhab/Ha and 350 housing/ha [420 inhab/Ha and 220 housing/Ha gross density]. Therefore, the densities range proposed by AEUB stands as *reasonable*.

— in international legislation, we have found the maximum value of population density in the City of Buenos Aires, with a maximum limit of 1,000 inhab/Ha[20]

The fourth approach is based on compliance with free space standards. Hernández Aja et al. (1997) proposes 10 sq.m road + 10 sq.m green area per inhabitant for central areas. If we establish a minimum road and green zone per inhabitant area of 5 sq.m/inhab as a high dense situation, we achieve 1,000 inhab/Ha as unsustainability threshold. The fifth is from the *streets section*.

If the maximum ratio building/street section is 3:1 [H:W], then for a 100 x 100 m grid with 20m wide streets we obtain 6,400 sq.m plots [80 x 80 m]. Assuming 60 m height buildings [20 floors] and 18 m built depth, we obtain 25,560 sq.m built per plot. Considering 100 sq.m/housing would result 255 houses * 2.5 hab/viv implying 639 inhab/Ha.

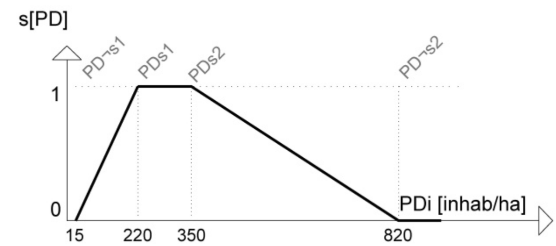
We see different criteria lead us to quite different figures of upper density limit, and the absence of a criterion that makes an assessment/perspective more important than the others makes us propose an *unsustainability threshold of 820 inhab/ha, approximately the average of the values obtained through the different approaches.*

Therefore, we establish the following sustainability / unsustainability limits for the Population Density indicator, PD:

— unsustainability limits. We adopt values proposed by AEUB (2010): PD_{s1} = 220 inhab/Ha and PD_{s2} = 350 inhab/Ha;

— unsustainability thresholds. We use above explained values: $PD_{\neg s1}$ = 15 inhab/Ha and $PD_{\neg s2}$ = 820 inhab/Ha.

The graph and formula of the sustainability function are:

Graphic Representation	Indicator Formalization
	$s[PD] = \max\left[\min\left[\frac{\mathrm{PD_i} - PD_{\neg s1}}{\mathrm{PD_{s1}} - DP_{\neg s1}}; 1; 1 - \frac{PD_i - PD_{s2}}{PD_{\neg s2} - DP_{s2}}\right]; 0\right]$ Alternatively: $s[PD] = max\left[min\left[\frac{PD_i - 15}{220 - 15}; 1; 1 - \frac{PD_i - 350}{820 - 350}\right]; 0\right]$

Tab.3 Indicator for measuring the sustainability of Population Density

Therefore, to calculate the indicator, we first calculate the Population Density of the area using the formula:

$$PD_i = \frac{\mathrm{N}}{\mathrm{S}} \quad (2)$$

Being: PD_i Population Density [in persons/ha]; N_ Number of inhabitants and S_ Total gross surface of the urban area [Ha]

From above Population Density value PD_i we calculate the indicator as:

$$s[PD] = max\left[min\left[\frac{PD_i - 15}{205}; 1; 1 - \frac{PD_i - 350}{470}\right]; 0\right] \quad (3)$$

Futher comments.

The proposed indicator seeks to assess the sustainability of urban population density in neighborhood type areas (i.e., from 16-25 to 50 Ha surface) in a developed-country city model, with a 20m or more street width network. Noteworthy, there are issues that may require reducing above suggested limits in some cases:

[20] Código de Ordenamiento del territorio del Partido de General Pueyrredón, Buenos Aires. Art. 4.1.4.a. Maximum Net Population Density admissible values.

- the relation between maximum population and available residential area, which leads us to place the optimum population density in 2-3 hab/100 built sqm ratios, with a maximum threshold in a density somewhat lower than 4 inhab/100 built sq.m[21];
- the relation between optimal population density and streets width, which means that admissible/optimal densities are lower in urban networks with narrow streets[22];
- the existing relation between residential density and spatial segregation by income (Alvira, 2017a; Leal et al., 2012). Optimal densities explained above usually imply high spatial integration of different income residents, provided other related issues are adequate (green zones provision; street network functionality; urban scenery; housing area ratio per capita ...);
- the relation between optimal density values and the dimension of reviewed area means that suggested values should be lowered when assessing whole cities, neighborhoods of small towns or villages[23].

In addition, optimal population density values should be compatible with the morphological differentiation of cities areas, which requires admitting a sufficiently wide optimal density range excluding unsustainable morphologies.

Thus, herein proposed indicator assigns an acceptable sustainability value to a variety of urban morphologies, but urban morphologies implying lower than 110 inhab/Ha[24] or higher than 625 inhab/Ha population densities achieve lower than 0.5 indicator values. These densities stand as the thresholds from which population density starts to be more unsustainable than sustainable. Also, the increasing need for adaptation to climate change suggests herein proposed unsustainability thresholds could have to further approach the sustainable range (i.e., increasing $PD_{\neg s1}$ and decreasing $PD_{\neg s2}$).

The high energy and land consumption (increase in CO_2 emissions and reduction of agricultural land and biodiversity] coupled to low density values[25] as well as the overcrowding and high energy consumption (heat island effect, congestion, increase in air conditioning use…) coupled to excessively dense urban areas, may be increasingly unsustainable as the clime effectively changes, reducing thus the sustainability range.

Lastly, it is most likely that in many developed countries a large part of their territory whose urbanization is sustainable has already been urbanized[26], so territory for urban use stands as an increasingly scarce resource worldwide. This highlights the need to complete *population density* assessment with measures preventing urban land underutilization; more specifically, regulations that limit the construction of second residences and vacation homes. Besides, urban developments or cities densification should be planned and designed to

[21] Madrid City central area shows negative correlation [-0.57] between housing density and housing built area per capita (own calculation based on Madrid City Council and Cadaster data), which means an increase in the population density usually implies a reduction in per capita housing surface. Thus, when high densities are detected, it is necessary to monitor the per capita housing area. Nevertheless, it is worth highlighting overcrowding in cities is not as much linked to high population density as to homes overcrowding. To detect it Jacobs (1961) proposed assessing the number of people per room. JSBC (2011) assesses the ratio of housing area per inhabitant, suggesting a lousy situation if less than 28 sq.m/inhab and optimal if equal or greater than 40.5 sq.m/inhab [cities] or 47 sq.m/inhab (villages). It is interesting that Gómez-Piovano and Mesa (2017) find that the average per capita housing area in Mendoza Metropolitan Area is approx. 50 sqm/inhab but in lower income areas it reduces to 10 sq.m/inhab.

[22] Gómez-Piovano and Mesa (2017) calculate different recommended maximum densities to achieve good sunlight of the city in the Metropolitan Area of Mendoza (Argentina), which they relate to streets width. The authors suggest a range from 80 inhab/Ha for 10m wide streets to 395 inhab/Ha for streets wider than 19 m.

[23] Higueras (2009) suggests 100 housing/Ha as maximum admissible value to prevent congestion (between 250 and 300 inhab/Ha). OMAU (2012) suggests a minimum/desirable level of 120 inhab/Ha for a group of Mediterranean cities, stating that the optimum density value depends on the context.

[24] An area of semi-detached housing with 45 housing/ha, provides a population density of 112 inhab/ha for an average occupation of 2.5 persons/viv. Calthorpe Associates (2011) calculate water and energy consumption according to type of housing (detached houses big size; detached houses small size; townhouses and collective dwelling), obtaining that the consumption of an isolated detached house is between two and three times higher than that of a collective dwelling. According to own calculations (Alvira, 2017a) only row houses/townhouses and collective dwellings are below current thresholds for sustainable energy and water consumption.

[25] Güneralp et al. (2017), find urban density has similar (sometimes higher) impact for reducing energy consumption in cities than buildings energy efficiency. Energy savings are both linked to lower consumption in collective than isolate housing and to smaller housing surface, requiring lower energy for heating or cooling.

[26] This hypothesis has been tested in Spain, where at least 80% of sustainable urban territory according to Ecological Foot standards is already built up (Alvira, 2017a).

maximize resilience and adaptation to climate change (e.g., densification of current urban areas near the sea and close to sea level should be avoided) (Dodman, 2009).

5 CONCLUSIONS

The present article explains an easy methodology for formulating sustainability indicators within the framework of fuzzy logic/fuzzy sets theory. Building on this framework provides us several advantages compared to the usual *intuitive design* of indicators; both in the indicators formulation / design phase as well as in their subsequent testing[27]. Specifically, herein proposed methodology:

- it allows conceptualizing urban *sustainability assessment indexes* as functions that define the grade of membership of each city or urban area to Sustainability class [S] linking it to its non-membership to Unsustainability class [¬S];
- it allows conceptualizing *urban sustainability indicators* as functions that define the grade of membership of each urban area to each of the S_i subclasses implicit in class S; i.e., subclasses to which the city must have some membership to be able to have some membership to S[28];
- it provides a criterion to select which are the *relevant variables* that should be assessed in each indicator, those that can modify the membership of the city to subclass S_i, which is measured the indicator;
- it provides a criterion to define the *sustainability and unsustainability limits* for the relevant variables, as well as for the mathematical modeling of each indicator.

It is important to insist that the above four issues are criteria for both indicators formulation as well as for their testing and possible refutation or confirmation:

- *sustainability assessment indexes* should meet above condition; if a model does not properly -and simultaneously- characterize membership of the city to S and ¬S classes, then it is not a sustainability index (though it may be assessing another quality or urban phenomenon);
- *sustainability indicators* should satisfy above definition: if an indicator is not an adequate membership function to some class S_i necessary for sustainability, then it is not a sustainability indicator;
- the *relevant variables for each indicator* should satisfy above definition; if the relevant variables (it might be an aggregated variable) do not adequately characterize the city membership to class S_i assessed by the indicator, then they are not the relevant variables (or there are other relevant variables that also need to be valued);
- the *sustainability limits for the relevant variables* must delimit the range of values beyond which variations of the value of the variable do not modify the sustainability of the system and the membership function must adequately model the transition between said values.

Therefore, herein proposed methodology allows us to simplify and clarify -but also to systematize- the design of urban sustainability indicators. It facilitates communication to the rest of the scientific community of the premises on which each indicator is built. And it allows empirical test (both by the person who formulates the proposal and by other scientists).

These are three fundamental issues to optimize research in Sustainability and a requisite to effectively confront the urgent need to reduce the extremely high unsustainability of our cities and societies. Additionally, it is necessary to emphasize that urban sustainability should be assessed in an integrated manner so it can be

[27] For example of indicators testing, see Alvira (2017a & b).

[28] Conceptualizing Sustainability and Unsustainability as complementary classes S and ¬S also makes it easier to detect which are the qualities (subclasses S_i) that maximize the membership of a city to class S (both in terms of the city's probability of enduring and the degree to which its state is optimal) in terms of opposites. If it is possible to determine the qualities that make a city unsustainable (i.e., which imply its membership to ¬S), then it is possible to determine the issues that make it sustainable, which are the opposite. This facilitates detecting some relevant issues for sustainability which are difficult to detect as membership to class S, yet easy to detect in terms of membership to class ¬S. For details of the procedure to design a complete assessment model as well as criteria to check the completeness of the models, refer to Alvira (2014).

used as objective criteria for decision making regarding possible urban transformations. Furthermore, the urgency of reducing our cities unsustainability requires incorporating sustainability as the key decision making criterion when designing their transformations, which constitute their long term evolution. A rational society should be deemed as that which seeks to maximize its sustainability in all its decision making processes[29]. This implies that besides designing sustainability indicators, it is necessary to define their organization in models that should incorporate different levels linked to indicators structure of aggregation, and to define procedures so they can be used in most important decision making processes in our cities, which not only involve new urban projects, but also the modification of current legislation[30].

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[29] Needless to say, in an interconnected world, sustainability of different cities and societies is interconnected. Maximizing one's sustainability often requires maximizing the sustainability of other cities/societies. In order to achieve it global agreements are required, an issue which largely exceeds the present proposal. For an approach on how a global agreement could be designed see Alvira (2017a).

[30] In order to do so, the author proposed in 2014 a general axiomatic framework that provides a guideline for the design of models to quantitatively assess sustainability. In terms of operational models (i.e., models which can be used for decision making), the author has proposed a complete model which enables using sustainability maximization as a decision criterion in most urban transformations (Alvira, 2017a). For an application of this model to draft a Rooftop Code for a neighborhood of Madrid city (Alvira, 2016).

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IMAGE SOURCES

Fig. Cover: Monica Vilhelm & Ricardo Alvira.

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AUTHOR'S PROFILE

Ricardo Alvira Baeza is an architect and Urban Designer. From 2000 to 2010 he has worked in several major architectural firms, focusing on medium-large projects including: design and construction of a residential neighborhood; skyscrapers and retail centers; a university campus [this last project was designed seeking compliance with BREEAM and LEED ND sustainability criteria]. After 2010, he has focused on research, achieving a DEA in Urban Design and Planning [Polytechnic University of Madrid], with a study comparing the two major sustainability certification systems at the moment: LEED ND and BREEAM for Communities. His PhD Thesis in Architecture and Urban Planning [Polytechnic University of Cartagena], is a complete mathematical model for measuring cities sustainability [Meta_S] including a procedure so it can be used in most urban transformations [both urban projects and legislation drafting, …].